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# Nonlinear modeling of carbon dynamics in soil treated with tannery sludge

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## Abstract

In 2020, the Brazilian commercial cattle herd was the largest in the world, representing 14.3% of the worldwide herd, with 201.7 million heads. Although this activity yields significant profits, contributing to the economic and social development of Brazil, it has been the target of concerns, mainly due to the large production of waste/effluents associated to bovine leather processing. In this scenario, mineralization is important because nutrients essential for plant growth are released during the process of organic waste decomposition, and the dynamics of carbon release can be described by nonlinear regression models. Thus, this study aimed to model the mineralization of organic carbon in the soil for 6, 12, 24, and 36 megagrams doses per hectare (Mg ha$^{-1}$) of tannery sludge using the Stanford & Smith, Cabrera, and Juma nonlinear models. Very clayey soil samples were used: eutroferric red nitosol (NVef). Mineralized carbon was measured in 21 observations over time until the 105th day of incubation. Parameters were estimated using the least squares method. Adjustments were compared using the corrected Akaike information criterion (AICc) and the adjusted coefficient of determination ($R^2_{aj}$) as selection criteria. The Cabrera model had the best adjustments for doses 6, 12, and 24 Mg ha$^{-1}$ and Juma for dose 36 Mg ha$^{-1}$, based on the selection criteria used. Although the Stanford & Smith model is the most widely used in the literature to model soil carbon dynamics, its use was not the most appropriate for any of the doses evaluated in this study. The higher the dose of tannery sludge, the greater the amount of potentially mineralizable carbon.

**Keywords:** Stanford & Smith, Cabrera, Juma, Regression.

## Introduction

Tanning is the process of turning animal skin, which is a decomposing raw material, into leather (BRITO, 2013). According to the Center for the Brazilian Tanning Industry (CICB, 2019), Brazil exported, in 2019, about 181.9 million $m^2$ of tannage. One hide is estimated to produce about 15 kg of solid waste, and of these, 7.5 kg decant as sludge at the bottom of the primary treatment lagoon (KONRAD, CASTILHOS, 2002).

Currently, the most used method to discard tannery waste (sludge) is disposing it in dumps or landfills, which represents a high risk due to the accumulation and concentration of potentially toxic materials, such as phenols, sulfides, sodium and chromium, which can leach out and contaminate aquifers (KONRAD, CASTILHO, 2002). According to Brazil's environmental regulation, the tanning industry is a potential polluter and must dispose of its waste following state and/or federal norms, within the environmental quality standards established by law (BRASIL, 1981).

However, the economic relevance of this industry, combined with its polluting potential, has stimulated studies to broaden their knowledge about possible alternatives for disposal or reuse of sludge (MARTINES et al., 2006). According to Martines et al. (2006), the agronomic use of sludge is one of these alternatives, mainly because it promotes corrective actions and soil fertilization. Its use for soil nutrition is also a widely used alternative, highlighting the application of technical criteria for this type of waste in agriculture.

Although it is rich in macro- and micronutrients, understanding the dynamics of tannery sludge decomposition in soil is necessary to properly dispose of it in the environment. According to Feitosa et al. (2015), the addition of an organic waste to the soil stimulates microbial activity and the microorganisms use carbon as an energy source for their metabolic processes. Thus, part of the carbon is incorporated into microbial cells and part is released as $CO_2$.

According to Youngquist (2017), these microorganisms break down complex carbon-based molecules and, as a result, nutrients are made available to the plants and carbon dioxide is released as a byproduct. Moreover, the author states that microbial activity leads to increased rates of organic matter metabolism in soil and subsequent loss of carbon with $CO_2$. Pulrolnik (2019) stresses that at the beginning of organic waste decomposition in soil, the amount of mineralized carbon is higher due to the presence of easily degradable substance fractions, and then this amount reduces, as more resistant carbon fractions are mineralized. This process has been described by nonlinear regression models, since they provide adequate adjustments (PAULA et al., 2019; SILVA et al., 2019a,b) and their parameters allow practical interpretation (MIRANDA et al., 2021; JANE et al., 2020a; SILVA et al., 2020c; SILVA et al., 2021a).

Paula et al. (2019) and Manzoni and Porporato (2007) highlight the importance of studying carbon mineralization curves and knowing the statistical model that describes the processes as a function of time, considering the need to understand the dynamics of organic waste decomposition, which can help in the soil management practices more favorable to agricultural crop production. Paula (2012) also highlights that the statistical model enables the estimation of the amounts of waste recommended for application to the soil.

Among the most widely used models to describe carbon mineralization in organic waste decomposition, the Stanford & Smith (1972), Cabrera (1993), and Juma (1984) models stand out. The Stanford & Smith and Juma models consider that waste has a potentially mineralizable amount of carbon. On the other hand, the Cabrera model considers that waste has some amounts that are easily mineralizable and another that is constantly mineralized (PAULA et al., 2020; SILVA et al., 2020a; VILELA et al., 2022). These models are widely used by many authors and have adequate adjustments. In the study by Silva et al. (2019a), they adjusted models in the decomposition of sewage sludge and oat straw, and the study by Paula et al. (2019) compared nonlinear models in the description of carbon mineralization in soil treated with swine manure.

Thus, this study aims to adjust and compare the Stanford & Smith (1972), Cabrera (1993), and Juma (1984) nonlinear regression models in describing carbon dynamics in an eutroferric red nitsoil treated with different doses of tannery sludge.

## Material and methods

The data used to adjust the models were extracted from Martines et al. (2006) and correspond to an experiment that evaluated the mineralization of organic carbon from tannery sludge applied to a soil (0–20 cm depth) of very clayey texture: eutroferric red nitosol (NVef).

The tannery sludge used in the experiment was a 1:1 mixture of the liming sludge produced in the liming stage and the primary sludge from an effluent treatment plant (ETP), resulting from the precipitation of the effluents generated in the process, except for the effluents containing chromium. This sludge was collected at the Curtume Vanzella factory, located in the municipality of Rolândia, Paraná. The sludge doses used in the NVef soil (very clayey) were 6, 12, 24, and 36 Mg ha$^{-1}$ (dry base), considering 20 cm depth and a soil density of 1 g $cm^{-3}$.

The experimental design was completely randomized, in plots subdivided in time (five doses and 21 evaluation times), with three replications. Each sludge dose was applied to 200 g of soil and they were evaluated at one, two, three, four, five, six, seven, eight, nine, 10, 11, 12, 13, 14, 15, 20, 25, 40, 60, 80, and 105 days of incubation. To analyze and model the organic carbon mineralization of tannery sludge doses the nonlinear regression models of Stanford & Smith (1) and Cabrera (2), reparameterized by Zeviani et al. (2012), and Juma (3) were used.

$$C_i = C_0\left(1 - e^{\frac{-\ln(2)t_i}{v}}\right) + \varepsilon_i \qquad (1)$$

$$C_i = C_1\left(1 - e^{\frac{-\ln(2)t_i}{v_1}}\right) + k_0 t_i + \varepsilon_i \qquad (2)$$

$$C_i = \frac{C_0 t_i}{v + t_i} + \varepsilon_i \qquad (3)$$

In Equations (1), (2), and (3), $C_i$ is the observed value of mineralized carbon after adding the tannery sludge dose in time $t_i$; $C_0$ is the potentially mineralizable carbon of the tannery sludge dose; $k_0$ is the mineralization rate or constant; $v$ and $v_1$ represent the half-life of potentially mineralizable and easily mineralizable carbon, respectively, corresponding to the time required for half of the organic carbon to be mineralized; and $\varepsilon_i$ is the deviation, which is assumed to have normal distribution with mean 0 and constant variance $\sigma^2$, i.e., $\varepsilon_i \sim N(0,\sigma^2)$.

The models were adjusted using the ordinary least squares and the Gauss-Newton iterative methods, since the system of normal equations has no explicit solution (RIBEIRO et al., 2018b; SILVA et al., 2020b; FERNANDES et al., 2022b; FRÜHAUF et al., 2022a,b). To verify the assumptions of the regression models, the Shapiro-Wilk, Durbin-Watson, and Breusch-Pagan tests were used to assess whether the residuals of the models were normally distributed, independent, and homoscedastic, respectively.

When the residuals were autocorrelated, the model was adjusted using the generalized least squares method, incorporating the first-order autoregressive parameter AR(1). In the case of heterogeneous residual variances, the adjustment was performed using the weighted least squares method, as the estimation process incorporates the existing uncertainty regarding each factor (FERNANDES et al., 2014). When the normality assumption was met, the 95% confidence interval of the model parameters was constructed (DRAPER, SMITH, 2014).

The corrected Akaike information criterion (AICc) and the adjusted coefficient of determination ($R^2_{aj}$) were used to select the model that best describes the organic carbon mineralization of the tannery sludge doses.

R programming language (R DEVELOPMENT CORE TEAM, 2022) was used to estimate the model parameters, perform residual analysis tests, obtain the selection criteria values, and prepare graphs.

## Results and discussion

Initially, the Stanford & Smith, Cabrera, and Juma nonlinear models were adjusted to the organic carbon mineralization data of tannery sludge doses, and the assumptions about the error vector of the models were analyzed — that is, whether the residuals were independent and identically distributed, following a normal distribution with zero mean and constant variance, $\varepsilon_i \sim N(0,\sigma^2)$. We obtained the results of the residue analysis (Table 1) using the Shapiro-Wilk, Breusch-Pagan, and Durbin-Watson tests. Several authors used these tests to verify the assumptions of regression models (SILVA et al., 2021b; JANE et al., 2020b; PRADO et al., 2020; FRÜHAUF et al., 2020; JANE et al., 2019).

**Table 1.** P-values of the Shapiro-Wilk, Durbin-Watson and Breusch-Pagan tests applied to the residuals of the Stanford & Smith, Cabrera, and Juma models for doses 6, 12, 24, and 36 Mg ha$^{-1}$ of tannery sludge.

| Treatment | Model | Shapiro-Wilk *p-value* | Durbin-Watson *p-value* | Breusch-Pagan *p-value* |
|---|---|---|---|---|
| Dose 6 | Stanford & Smith | 0.6913 | <0.01* | 0.0228 |
| | Cabrera | 0.0699 | <0.01* | 0.0611 |
| | Juma | 0.5906 | <0.01* | 0.0190 |
| Dose 12 | Stanford & Smith | 0.2320 | <0.01* | 0.0527 |
| | Cabrera | 0.0451 | <0.01* | 0.0667 |
| | Juma | 0.0576 | <0.01* | 0.0345 |
| Dose 24 | Stanford & Smith | 0.3887 | <0.01* | 0.0323 |
| | Cabrera | 0.0493 | <0.01* | 0.0626 |
| | Juma | 0.5870 | <0.01* | 0.0122 |
| Dose 36 | Stanford & Smith | 0.7303 | <0.01* | 0.0122 |
| | Cabrera | 0.3473 | <0.01* | <0.01* |
| | Juma | 0.9421 | <0.01* | <0.01* |

**Source:** Prepared by the authors (2022).

For all doses, all models met the assumption of residual normality (Table 1) at the 1% significance level (p>0.01) in the Shapiro-Wilk test. In the Breusch-Pagan test, all models had homogeneous residual variances for doses 6, 12, and 24 Mg ha$^{-1}$ of tannery sludge (Table 1).

For dose 36 Mg ha$^{-1}$, the Cabrera and Juma models violated the assumption of homogeneity of residuals (p<0.01), thus, the parameters were estimated in a weighted manner (FERNANDES et al., 2014) for these models, adjusted for the 36 Mg ha$^{-1}$ dose.

In the Durbin-Watson test, considering the 1% significance level, the hypothesis of independence of residuals was not met (p<0.01), showing that the residuals were correlated in all models for all doses, which was expected, since the data were obtained over time in the same plot (FERNANDES et al., 2022a; PEREIRA et al., 2022). Paula et al. (2019) also observed autocorrelation in the adjustment of nonlinear models to carbon mineralization data in soil treated with swine manure. Moreover, Silveira et al. (2018) observed autocorrelation of errors when adjusting the nonlinear model to the cumulative biogas production of swine waste.

Due to the residual dependence, the Stanford & Smith, Cabrera, and Juma models were adjusted by the generalized least squares method, incorporating the first-order autoregressive parameter AR(1), since, according to Ribeiro et al. (2007) and Pereira et al. (2022), in the presence of correlated residuals, modeling this correlation and incorporating it into the model is important to ensure higher precision in the estimates and better quality in the adjustment.

Table 2 presents the estimates of the parameters and their respective confidence intervals for the Stanford & Smith, Cabrera, and Juma models for dose 6 Mg ha$^{-1}$ in the NVef soil.

According to Table 2, in the Stanford & Smith model, the estimate for potentially mineralizable carbon for dose 6 Mg ha$^{-1}$ was 103.59 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of five days. In the Cabrera model, the estimate for easily mineralizable carbon for dose 6 Mg ha$^{-1}$ was 66.09 C-CO2 mg 100 g$^{-1}$ soil with a half-life of four days. In the Juma model, the estimate of potentially mineralizable carbon was 120.69 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of 12 days. We estimated potentially

**Table 2.** Estimation of the parameters of the Stanford & Smith, Cabrera, and Juma models and their respective asymptotic 95% confidence intervals (LL: lower limit; UL: upper limit) in the adjustment of C-$CO_2$ mg 100 g$^{-1}$ soil for 6 Mg ha$^{-1}$ dose of tannery sludge.

| Stanford & Smith | | | | Cabrera | | | | Juma | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | LL | Estimate | UL | | LL | Estimate | UL | | LL | Estimate | UL |
| $C_0$ | 63.9701 | 103.5980 | 143.2259 | $C_1$ | 59.3417 | 66.0958 | 72.8498 | $C_0$ | 109.1738 | 120.6946 | 132.2154 |
| Ø | 0.7872 | 0.9607 | 0.9932 | $k_0$ | 0.0049 | 0.0065 | 0.0081 | Ø | 0.5456 | 0.8720 | 0.9687 |
| $v$ | 4 | 5 | 8 | Ø | 0.4969 | 0.8405 | 0.9563 | $v$ | 8 | 12 | 15 |
| | | | | $v_1$ | 3 | 4 | 5 | | | | |

**Source:** Prepared by the authors (2022).

mineralizable carbon for different amounts by the Stanford & Smith and Juma models, and, according to Zeviani et al. (2012), selecting the most appropriate model to describe this process is key to make inferences about this amount.

Table 3 shows the results of the selection criteria of the models. Among the results found for dose 6 Mg ha$^{-1}$, the Cabrera model had the highest $R^2_{aj}$ and the lowest AICc. Based on the lowest AICc value, the Cabrera model had the best adjustment. Thus, this dose has mineralizable carbon fractions with exponential behavior and more resistant fractions, with constant mineralization (SILVA et al., 2019b; PAULA et al., 2020).

Figure 1 shows the mineralized carbon of dose 6 Mg ha$^{-1}$ of tannery sludge and the adjustments of the Stanford & Smith, Cabrera, and Juma models in the NVef soil. According to Table 3, the $R^2_{aj}$ values of the Cabrera and Juma models were greater than 97%, showing that the models were well adjusted, in line with Figure 1. For the Stanford & Smith model, $R^2_{aj}$ was 68.88% (Table 3).

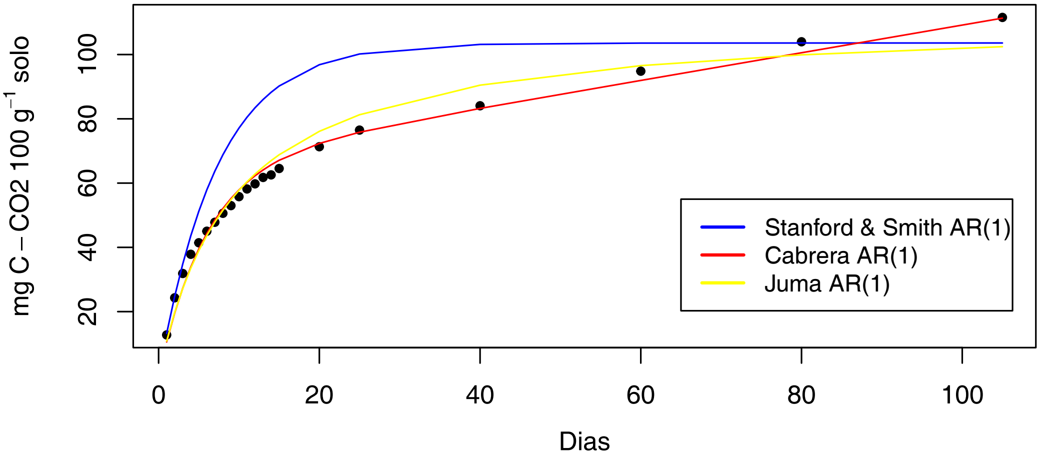
Table 4 presents the estimates of the parameters and their respective confidence intervals for the Stanford & Smith, Cabrera, and Juma models for dose 12 Mg ha$^{-1}$ in the NVef soil. In the Stanford & Smith model, the estimate for potentially mineralizable carbon for dose

**Table 3.** Selection criteria of the Stanford & Smith, Cabrera, and Juma models for tannery sludge doses in Mg ha$^{-1}$.

| Dose (Mg ha$^{-1}$) | Model | Selection criteria | |
|---|---|---|---|
| | | $R^2_{aj}$ | $AIC_c$ |
| 6 | Stanford & Smith | 0.6888 | 114.5913 |
| | Cabrera | 0.9891 | 83.0478 |
| | Juma | 0.9773 | 100.2795 |
| 12 | Stanford & Smith | 0.9532 | 124.9100 |
| | Cabrera | 0.9841 | 92.2283 |
| | Juma | 0.9837 | 99.4943 |
| 24 | Stanford & Smith | 0.9644 | 134.6737 |
| | Cabrera | 0.9908 | 100.7563 |
| | Juma | 0.9911 | 105.4970 |
| 36 | Stanford & Smith | 0.9696 | 145.0586 |
| | Cabrera | 0.9947 | 113.4886 |
| | Juma | 0.9935 | 112.0194 |

**Source:** Prepared by the authors (2022).

**Figure 1.** Adjustments of the Stanford & Smith, Cabrera, and Juma models for the carbon mineralization of dose 6 Mg ha$^{-1}$ of tannery sludge in the NVef soil.




**Source:** Prepared by the authors (2022).

12 Mg ha$^{-1}$ was 133.53 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of eight days. In the Cabrera model, the estimate for easily mineralizable carbon was 96.83 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of five days. In the Juma model, the estimate of potentially mineralizable carbon was 152.89 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of nine days.

According to Table 3, for dose 12 Mg ha$^{-1}$, the Cabrera model had the highest $R^2_{aj}$ and the lowest AICc. Therefore, the Cabrera model had the best adjustment for dose 12 Mg ha$^{-1}$ of tannery sludge in the NVef soil. Figure 2 shows the mineralized carbon of dose 12 Mg ha$^{-1}$ of tannery sludge and the adjustments of the Stanford & Smith, Cabrera, and Juma models in the NVef soil. All models had adequate adjustment, since the $R^2_{aj}$ values of the adjusted models were above 95% (Table 3). In the description of the data of organic waste mineralization in soil, several authors obtained adequate adjustments with $R^2_{aj}$ values above 90% (PAULA et al., 2020; VILELA et al. 2022).

Table 5 presents the estimates of the parameters and their respective confidence intervals for the Stanford & Smith, Cabrera, and Juma models for dose 24 Mg ha$^{-1}$ in the NVef soil.

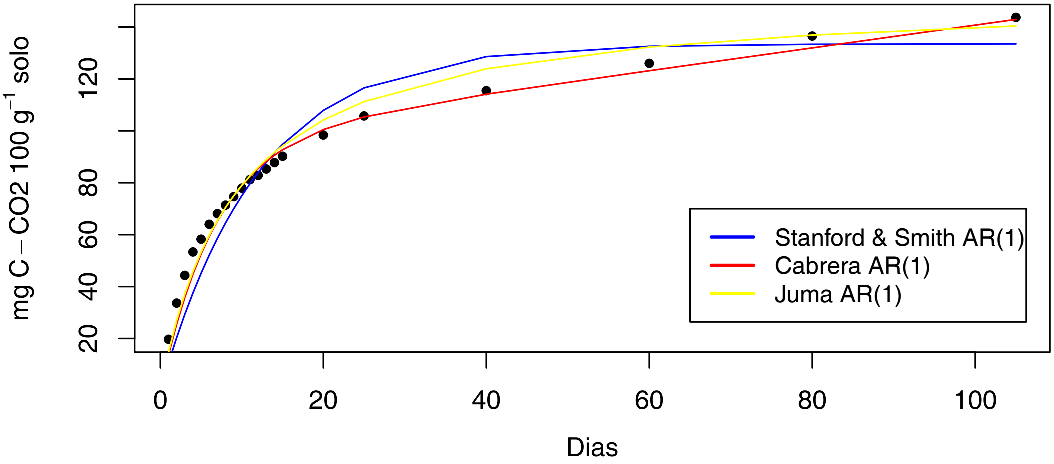
According to Table 5, the Stanford & Smith model obtained an estimate of potentially mineralizable carbon of 187.42 C-$CO_2$ mg 100 g$^{-1}$ soil for dose 24 Mg ha$^{-1}$, with a half-life of eight days. In the Cabrera model, the estimate for easily mineralizable

**Table 4.** Estimation of the parameters of the Stanford & Smith, Cabrera, and Juma models and their respective asymptotic 95% confidence intervals (LL: lower limit; UL: upper limit) in the adjustment of C-$CO_2$ mg 100 g$^{-1}$ soil for dose 12 Mg ha$^{-1}$ of tannery sludge.

| Stanford & Smith | | | | Cabrera | | | | Juma | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | LL | Estimate | UL | | LL | Estimate | UL | | LL | Estimate | UL |
| $C_0$ | 114.2887 | 133.5320 | 152.7754 | $C_1$ | 85.8730 | 96.8380 | 107.8030 | $C_0$ | 141.1697 | 152.8998 | 164.6299 |
| Ø | 0.5863 | 0.9200 | 0.9867 | $k_0$ | 0.0032 | 0.0045 | 0.0058 | Ø | 0.5456 | 0.8720 | 0.9687 |
| $v$ | 7 | 8 | 12 | Ø | 0.6110 | 0.9286 | 0.9887 | $v$ | 7 | 9 | 12 |
| | | | | $v_1$ | 4 | 5 | 6 | | | | |

**Source:** Prepared by the authors (2022).

**Figure 2.** Adjustments of the Stanford & Smith, Cabrera, and Juma models for the carbon mineralization of dose 12 Mg ha$^{-1}$ of tannery sludge in the NVef soil.




**Source:** Prepared by the authors (2022).

**Table 5.** Estimation of the parameters of the Stanford & Smith, Cabrera, and Juma models and their respective asymptotic 95% confidence intervals (LL: lower limit; UL: upper limit) in the adjustment of C-$CO_2$ mg 100 g$^{-1}$ soil for dose 24 Mg ha$^{-1}$ of tannery sludge.

| Stanford & Smith | | | | Cabrera | | | | Juma | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | LL | Estimate | UL | | LL | Estimate | UL | | LL | Estimate | UL |
| $C_0$ | 165.1789 | 187.4215 | 209.6641 | $C_1$ | 129.5103 | 141.4471 | 153.3839 | $C_0$ | 199.4941 | 211.1535 | 222.8130 |
| Ø | 0.5663 | 0.9030 | 0.9814 | $k_0$ | 0.0028 | 0.0039 | 0.0049 | Ø | 0.4945 | 0.8493 | 0.9614 |
| $v$ | 6 | 8 | 10 | Ø | 0.5593 | 0.8877 | 0.9752 | $v$ | 7 | 9 | 10 |
| | | | | $v_1$ | 4 | 5 | 6 | | | | |

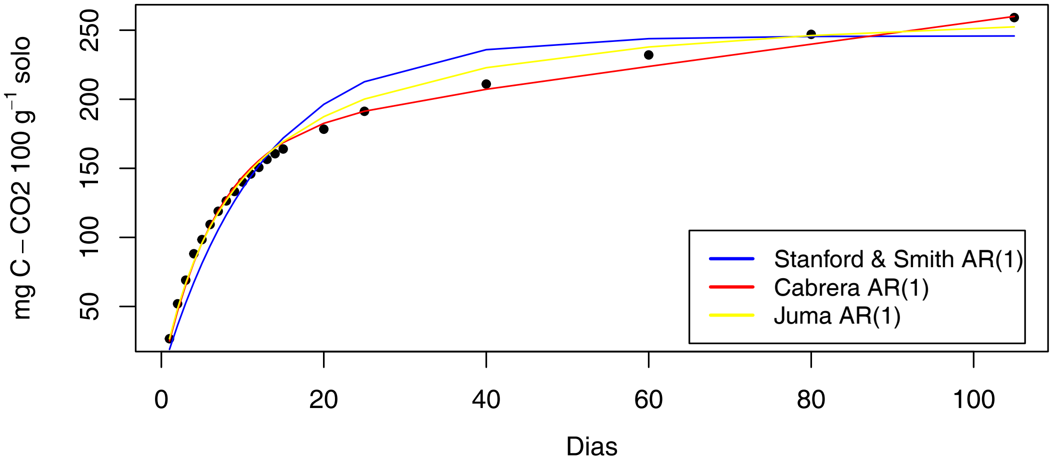
**Source:** Prepared by the authors (2022).

carbon was 141.44 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of five days. In the Juma model, the estimate of potentially mineralizable carbon was 211.15 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of nine days.

Based on the lowest AICc value (Table 3), the Cabrera model had the best adjustment for dose 24 Mg ha$^{-1}$ of tannery sludge. This model also had the best adjustment in a treatment in the study by Paula et al. (2019), which evaluated the description of mineralization in soil treated with swine manure. This dose, as well as the other aforementioned doses, has mineralizable carbon fractions with exponential behavior and more resistant fractions with constant mineralization. Figure 3 shows the mineralized carbon of dose 24 Mg ha$^{-1}$ of tannery sludge and the adequate adjustments obtained by the Stanford & Smith, Cabrera, and Juma models in the NVef soil. The $R^2_{aj}$ values of the models were above 96% (Table 3).

Table 6 presents the estimates of the parameters and their respective confidence intervals for the Stanford & Smith, Cabrera, and Juma models for dose 36 Mg ha$^{-1}$ in the NVef soil. In the Stanford & Smith model, the estimate for potentially mineralizable carbon was 245.87 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of nine days. In the Cabrera model, the estimate for easily mineralizable carbon was 175.50 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life of five days. In the Juma model, the estimate of potentially mineralizable carbon was 274.84 C-$CO_2$ mg 100 g$^{-1}$ soil with a half-life

**Figure 3.** Adjustments of the Stanford & Smith, Cabrera, and Juma models for the carbon mineralization of dose 24 Mg ha$^{-1}$ of tannery sludge in the NVef soil.




**Source:** Prepared by the authors (2022).

of nine days. Although the $C_0$ parameter had the same interpretation in the Juma and Stanford & Smith models, they estimate different amounts of potentially mineralizable carbon (Table 6) and, according to Sari et al. (2018), selecting the most appropriate model is key to understand the phenomenon studied.

According to Table 3, the Juma model had the lowest AICc value for dose 36 Mg ha$^{-1}$.This model had the best adjustment to the carbon mineralization data for this dose of tannery sludge in the NVef soil, which shows that this dose has only a fraction of mineralizable carbon. Figure 4 shows the mineralized carbon of dose 36 Mg ha$^{-1}$ of tannery sludge and the adjustments of the Stanford & Smith, Cabrera, and Juma models in the NVef soil.

In the adjustment of the Stanford & Smith, Cabrera, and Juma models for all doses (Tables 2, 4, 5, and 6), no confidence interval included the zero value, and, according to Zeviani et al. (2012), the models were adjusted to the treatments.

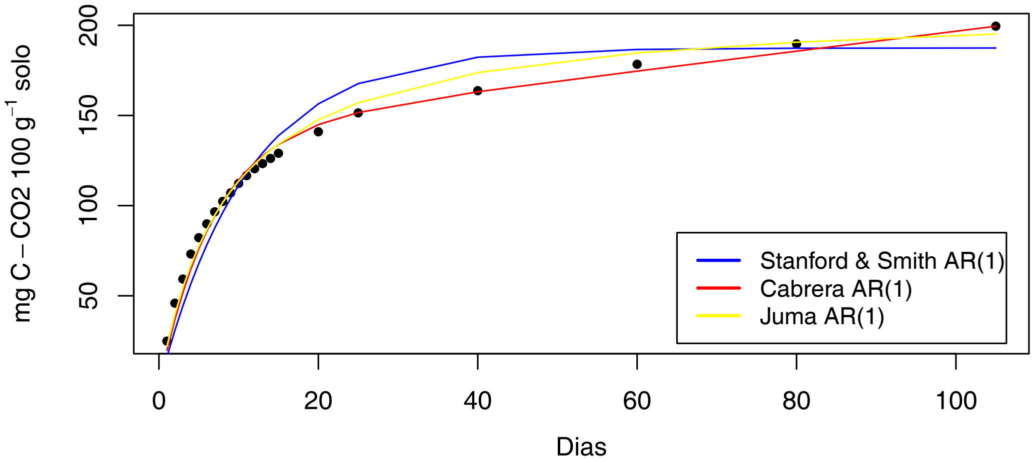
For doses 24 and 36 Mg ha$^{-1}$, the confidence intervals of the $C_0$ parameter did not overlap in the Stanford & Smith and Juma models (Tables 5 and 6). Thus, dose 36 Mg ha$^{-1}$ had the greatest potentially mineralizable carbon compared with all other adjusted doses. Increasing the dose stimulated microbial activity, enhancing the mineralization of the carbon added to the soil and the degradation of native soil organic matter, in line with Fernandes et al. (2011). Despite the difference in the amount

**Table 6.** Estimation of the parameters of the Stanford & Smith, Cabrera, and Juma models and their respective asymptotic 95% confidence intervals (LL: lower limit; UL: upper limit) in the adjustment of C-$CO_2$ mg 100 g$^{-1}$ soil for dose 36 Mg ha$^{-1}$ of tannery sludge.

| **Stanford & Smith** | | | | **Cabrera** | | | | **Juma** | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | LL | Estimate | UL | | LL | Estimate | UL | | LL | Estimate | UL |
| $C_0$ | 219.7508 | 245.8742 | 271.9976 | $C_1$ | 161.6583 | 175.5068 | 189.3552 | $C_0$ | 259.5319 | 274.8435 | 290.1551 |
| Ø | 0.5467 | 0.8819 | 0.9735 | $k_0$ | 0.0033 | 0.0045 | 0.0058 | Ø | 0.4025 | 0.7924 | 0.9389 |
| $v$ | 7 | 9 | 11 | Ø | 0.3848 | 0.7626 | 0.9215 | $v$ | 8 | 9 | 11 |
| | | | | $v_1$ | 4 | 5 | 6 | | | | |

**Source:** Prepared by the authors (2022).

**Figure 4.** Adjustments of the Stanford & Smith, Cabrera, and Juma models for the carbon mineralization of dose 36 Mg ha$^{-1}$ of tannery sludge in the NVef soil.




**Source:** Prepared by the authors (2022).

of potentially mineralizable carbon ($C_0$), for all doses in the Stanford & Smith model, the confidence intervals of the half-life (v) parameter overlapped, showing that all doses require about eight days to mineralize half of the potentially mineralizable carbon.

For the Cabrera model, the confidence intervals of the $C_1$ parameter did not overlap for doses 6, 12, 24, and 36 Mg ha$^{-1}$ (Tables 2, 4, 5, and 6). Thus, dose 36 Mg ha$^{-1}$ had greater easily mineralizable carbon. The half-life of the easily mineralizable carbon ($v_1$) overlapped for all doses. Moreover, all doses spend about five days to mineralize half of the easily mineralizable carbon.

Although the Stanford & Smith model is the most widely used to model organic waste mineralized carbon (MARTINES et al., 2006; NUNES et al., 2016; ANDRADE et al., 2016), Table 3 shows that it was not the most appropriate model in any situation compared with the Cabrera and Juma models, since the AICc value was higher in all cases. Thus, we based the inference for potentially mineralizable carbon ($C_0$) on the Juma model, as its confidence intervals for the $C_0$ parameter did not overlap for doses 6, 12, 24, and 36 Mg ha$^{-1}$ (Tables 2, 4, 5, and 6), showing that the higher the applied dose, the greater the amount of potentially mineralizable carbon. On the other hand, the confidence intervals of the half-life (v) parameter for all doses in the Juma model overlapped. Moreover, all doses spend about nine days to mineralize half of the potentially mineralizable carbon.

Regarding potentially mineralized carbon ($C_0$), in the study by Martines et al. (2006), for dose 36 Mg ha$^{-1}$ of tannery sludge, the estimated $C_0$ was 175 C-$CO_2$ mg 100 g$^{-1}$ soil. On the other hand, in our study, based on the Juma model, which had better adjustment, $C_0$ was 274 C-$CO_2$ mg 100 g$^{-1}$ soil. Paula et al. (2019), when describing the mineralized carbon in soil treated with swine manure for one of their doses, obtained the best adjustment using the Juma model.

## Conclusions

The Stanford & Smith, Cabrera, and Juma nonlinear models adjusted the carbon mineralization doses 6, 12, 24, and 36 Mg ha$^{-1}$ of tannery sludge in typical eutroferric red nitosol (NVef). The higher the dose of tannery sludge, the greater the amount of potentially mineralizable carbon.

Cabrera was the most appropriate model to describe the carbon mineralization of doses 6, 12, and 24 Mg ha$^{-1}$ of tannery sludge. For dose 36 Mg ha$^{-1}$, Juma was the most appropriate model. Although Stanford & Smith model is the most widely used in the literature to model soil carbon dynamics, its use was not the most appropriate in any of the doses evaluated in this study.

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