

Mean cumulative global irradiation at ground level described by sigmoidal models, in Belo Horizonte/MG

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Abstract

The variation of sunlight throughout the year, caused by the tilt of the Earth's axis and its spherical shape, influences surface heating, temperatures, atmospheric currents, and rainfall distribution. Solar energy, being clean and sustainable, has many applications. Using it efficiently requires understanding its temporal distribution. The daily accumulation of solar irradiation follows a sigmoidal pattern, which can be described by non-linear regression models. These models offer practical and innovative interpretations for renewable energy projects, filling a gap in the scientific literature. This study was conducted to evaluate the suitability of the Logistic, Gompertz, and Von Bertalanffy non-linear regression models to describe the accumulation of daily global solar irradiation (kJ m^{-2}). Data on global irradiation accumulated throughout the day at ground level, during the equinoxes and solstices in Belo Horizonte/MG, were obtained from the Brazilian National Institute of Meteorology (INMET) through the Annual Historical Data for the period 2018 to 2022. The Logistic and Gompertz models met the evaluated statistical assumptions, unlike the Von Bertalanffy model. The Logistic model achieved a better fit for the equinox and winter solstice data, while the Gompertz model was more suitable for the summer solstice data. The asymptotic estimates (α) of the Logistic model were statistically equivalent for the equinoxes but not for the solstices. The β parameter (abscissa of the inflection point) was statistically equivalent for the solstices and the spring equinox.

Keywords: Solar irradiation. Energy accumulation. Gompertz model. Logistic model. Points of maximum and minimum acceleration.

Introduction

The growing concern for environmental conservation, population growth, and evolving operations in the service and industrial sectors are driving demand toward large-scale renewable energy sources. The current challenge is to make energy available in sufficient quantity and with the necessary quality.

According to the Brazilian Energy Research Office (EPE, 2023), the global energy matrix in 2021 had the following distribution: coal (27.2 %), oil and derivatives (29.5 %), natural gas (23.6 %), hydropower (2.5 %), nuclear (5 %), biomass (9.5 %), and other sources (2.7 %). In Brazil, in 2022, the composition was different: oil and derivatives represented 37.5 % of the matrix, followed by sugarcane derivatives (15.4 %), hydropower (12.5 %), natural gas

(10.5 %), firewood and charcoal (9.0 %), wind and solar sources (3.5 %), other renewables (7.0 %), coal (4.6 %), nuclear (1.3 %), and other non-renewables (0.6 %). At the Brazilian national level, about 47.4 % of the energy matrix comes from renewable sources, while 84.8 % of electricity generation comes from these sources. Despite this, the share of solar energy is still relatively small, corresponding to 4.4 % (29.8 TWh) of the total. Considering the high levels of solar radiation in the country, this resource has great potential to expand its relevance in the Brazilian energy system.

Although the flux of solar energy reaching Earth's atmosphere is relatively constant, the amount that actually reaches the surface is influenced by several factors, such as the Earth's rotation and revolution, the tilt of its

axis, and atmospheric and climatic conditions. The oblate spheroid shape of the planet causes solar rays to strike at different angles depending on latitude: near the Equator, radiation arrives almost perpendicular to the ground, while in polar regions it reaches the surface at a very oblique angle. This variation in incidence is responsible for differences in the heating of the Earth's surface. Moreover, the tilt of the rotation axis relative to the orbital plane causes the alternation in the amount of sunlight received by the hemispheres during Earth's revolution around the Sun, a phenomenon that explains the seasons. At the equinoxes, however, there is a balance in illumination between the hemispheres (PAIVA, 2018; BEDAQUE, BRETONES, 2016).

The variation in the illuminated extent is related to light intensity (energy) and determines the seasons. The tilt of the rotation axis, combined with the Earth's spherical shape, therefore determines variations in heating both throughout the year at the same latitude and, at the same moment, between different latitudes. These variations result in different temperatures on the Earth's surface, which are responsible for atmospheric currents. These currents transport water vapor and are decisive for the distribution of clouds and precipitation across the planet (NEGRÃO, 2008).

Solar energy is a clean and sustainable source, employed in technological processes of heating, drying, and electricity generation. The versatility of this resource requires understanding of how it is distributed in time and space. The flux of global solar irradiation at ground level varies throughout the day, and the accumulation of this energy exhibits sigmoidal behavior, which can be described by non-linear regression models. Non-linear models have applications in various fields (FRÜHAUF *et al.*, 2022a; PAULA *et al.*, 2020; SILVA *et al.*, 2019b; SILVA *et al.*, 2020b; SILVA *et al.*, 2025; VILELA *et al.*, 2022) and are relevant due to the possibility

of practical interpretation of their parameters (PAULA *et al.*, 2019; SILVA *et al.*, 2019a; SILVA *et al.*, 2022). The proposed modeling is innovative for renewable energy projects, as there are no existing studies using this methodology to analyze solar irradiation accumulation, thus representing a significant contribution to the field of applied sciences.

Accordingly, this study was conducted to evaluate the suitability of the Logistic, Gompertz, and Von Bertalanffy non-linear regression models to describe the accumulation of daily global solar irradiation (kJ m^{-2}) at ground level, at the solstices and equinoxes, based on data from the Brazilian National Institute of Meteorology (INMET) for the years 2018 to 2022, in the city of Belo Horizonte/MG; and to study the critical points of the models.

Material and methods

Data on global irradiation accumulated throughout the day at ground level, during the equinoxes and solstices in Belo Horizonte/MG, were obtained from INMET through the Annual Historical Data for the period 2018 to 2022, collected at station A521, located at coordinates (-19.88390, -43.96940). This station is identified by the World Meteorological Organization (WMO) code, followed by its latitude and longitude. The data refer to the intensity of global solar irradiation (kJ m^{-2}), recorded at two-hour intervals and accumulated throughout the day, based on UTC (Coordinated Universal Time), and are available in INMET's Historical Meteorological Data (2023).

The Logistic (1), Gompertz (2), and Von Bertalanffy (3) models (RIBEIRO *et al.*, 2018; FRÜHAUF *et al.*, 2020; PRADO *et al.*, 2020; SOUZA *et al.*, 2014) were fitted to the cumulative global irradiation data (kJ m^{-2}) and the time of day (UTC):

$$Y_i = \frac{\alpha}{1 + e^{\kappa(\beta-x_i)}} + \varepsilon_i; \quad (1)$$

$$Y_i = \alpha e^{-e^{\kappa(\beta-x_i)}} + \varepsilon_i; \quad (2)$$

$$Y_i = \alpha \left[1 - \frac{e^{\kappa(\beta-x_i)}}{3} \right]^3 + \varepsilon_i; \quad (3)$$

The parameter “ α ” indicates the total accumulation of global irradiation in kJ m^{-2} , while “ β ” indicates the abscissa of the inflection point in “UTC”. The parameter “ κ ” indicates the evolution constant of global irradiation in “ UTC^{-1} ”. The random error ε_i is assumed to have a normal distribution with zero mean and constant variance σ^2 , i.e., $\varepsilon_i \sim N(0, \sigma^2)$, in which Y_i corresponds to the intensity of global irradiation accumulated up to hour x_i , between 8 UTC and 24 UTC. According to Fernandes *et al.* (2015), in this parameterization of the Logistic model, the inflection point occurs at the $(\beta, \frac{\alpha}{2})$ coordinate; in the Gompertz model, at $(\beta, \frac{\alpha}{e})$; and in the Von Bertalanffy model, at $(\beta, \alpha(\frac{2}{3})^3)$. Thus, the Logistic model is symmetric about the inflection point, while the Gompertz and Von Bertalanffy models show asymmetry of the curves at that point.

The abscissa of the point of maximum acceleration occurs, in the Logistic model, at $\beta + \frac{1}{\kappa} \ln(2 - \sqrt{3})$; in the Gompertz model, at $\beta + \frac{1}{\kappa} \ln(\frac{3-\sqrt{5}}{2})$; and in the Von Bertalanffy model, at $\beta + \frac{1}{\kappa} \ln(\frac{4-\sqrt{7}}{3})$. The abscissas of the points of minimum acceleration are, respectively, $\beta + \frac{1}{\kappa} \ln(2 + \sqrt{3})$, $\beta + \frac{1}{\kappa} \ln(\frac{3+\sqrt{5}}{2})$, and $\beta + \frac{1}{\kappa} \ln(\frac{4+\sqrt{7}}{3})$ (MACHADO, 2024).

To estimate the model parameters, the Gauss-Newton algorithm, also known as the linearization method, was used. It consists of using a Taylor series expansion to approximate the non-linear regression model in linear terms, thereby enabling the application of the least squares method to obtain the parameter estimates (MAZUCHELI, ACHCAR, 2002).

As with most iterative methods, the first step is to assign initial values to the parameter vector $\theta^0 = (\alpha^0, \beta^0, \kappa^0)$. These were obtained through visual analysis of the data and through the graphical interface for fitting non-linear regression models, developed with tools from the rpanel package of the free software R (R Core Team, 2025).

To verify the assumption of normality of the residuals, the Shapiro-Wilk normality test was used, whose null hypothesis is that the residuals follow a normal distribution with mean μ and variance σ^2 . To verify the assumption of homoscedasticity of residual variances, the Breusch-Pagan test was used, whose null hypothesis is the homogeneity of residual variances. To verify the independence of the residuals, the Durbin-Watson test was used, whose null hypothesis is the independence of the residuals (SILVEIRA *et al.*, 2018; JANE *et al.*, 2020; FRÜHAUF *et al.*, 2022a; SILVA *et al.*, 2021).

After fitting the models, the comparison and selection of the model that best described the data were carried out based on the coefficient of determination (R^2), the residual standard deviation (RSD), and the Akaike Information Criterion (AIC). The model considered most adequate was the one that yielded the highest R^2 value and the lowest RSD and AIC values.

Results and discussion

The results of the residual analysis for the data from the city of Belo Horizonte/MG are presented in Table 1. The Von Bertalanffy model did not meet the assumption of independence of residuals for the equinoxes and the winter solstice, nor the assumption of homogeneity of variances for the summer solstice data, as can be seen in Table 1 by the p-values less than 0.05. Therefore, the Von Bertalanffy model was not considered for inference (FRÜHAUF *et al.*, 2022b; PAULA

Table 1. Results of the Shapiro-Wilk (SW), Breusch-Pagan (BP), and Durbin-Watson (DW) tests for the data from the city of Belo Horizonte/MG.

Model	Autumn Equinox – March			Spring Equinox – September		
	SW	BP	DW	SW	BP	DW
Logistic	0.5479	0.5722	0.2850	0.6544	0.5987	0.1858
Gompertz	0.4487	0.1149	0.3364	0.6670	0.3637	0.2593
Von Bertalanffy	0.7459	0.0849	0.0222	0.8460	0.4297	0.0024
Model	Winter Solstice – June			Summer Solstice – December		
	SW	BP	DW	SW	BP	DW
Logistic	0.8442	0.4139	0.3107	0.9343	0.2862	0.1040
Gompertz	0.6939	0.1929	0.2944	0.7981	0.2348	0.4569
Von Bertalanffy	0.6953	0.5677	0.0010	0.8072	0.0453	0.2303

Source: authors (2023).

et al., 2020; SILVA *et al.*, 2019b; SILVA *et al.*, 2020a). In contrast, the Gompertz and Logistic models met all assumptions, as observed by the p-values greater than 0.05.

The Logistic model provided the best fit for the equinox and winter solstice data, while the Gompertz model was more adequate for the summer solstice data, as shown by the higher values of the Coefficient of Determination (R^2) and the lower values of Residual Standard Deviation (RSD) and Akaike Information Criterion (AIC) in Table 2 (JANE *et al.*, 2019; OLIVEIRA *et al.*, 2013).

Based on the model fit evaluators, the Gompertz model yielded the best results for the summer solstice, and the Logistic model best fits the equinox and winter solstice data.

According to Ferreira (2006; our translation), in Sete Lagoas/MG, a location situated 70 km from the state capital, global radiation is attenuated by approximately 55 % in the summer as it passes through the atmosphere. “From June to August, there is less cloud cover, mainly due to the arrival of cold fronts, which allow large masses of cold, dry air to remain for longer periods.” In summer, days are longer; however, the highest insolation values were found in winter. The highest attenuation percentage occurs in summer, around 55.6 %, with the presence of clouds—and consequently a higher incidence of water vapor and aerosols in the atmosphere—being the main agent reducing incident radiation. The lowest attenuations occur in winter, potentially reaching a 36.8 % reduction relative to the total incident at the top of the atmosphere. Specific

Table 2. Model fit evaluators for the city of Belo Horizonte/MG: Residual Standard Deviation (RSD), Coefficient of Determination (R^2), and Akaike Information Criterion (AIC).

Model	Autumn Equinox – March			Spring Equinox – September		
	RSD	R^2	AIC	RSD	R^2	AIC
Logistic	396.10	0.9980	122.65	328.20	0.9988	119.64
Gompertz	453.30	0.9975	124.81	419.10	0.9981	123.55
Model	Winter Solstice – June			Summer Solstice – December		
	RSD	R^2	AIC	RSD	R^2	AIC
Logistic	303.40	0.9986	118.38	680.00	0.9952	131.30
Gompertz	352.10	0.9982	120.76	429.00	0.9982	123.93

Source: authors (2023).

characteristics of the Minas Gerais Cerrado, such as mountainous relief and high altitude, also influence solar irradiation attenuation.

According to Ramos, Vianna, and Marin (2018), the mean annual global solar radiation in the southeastern region is $17.5 \text{ MJ m}^{-2} \text{ day}^{-1}$, and in Belo Horizonte it ranges between $17.21 \text{ MJ m}^{-2} \text{ day}^{-1}$ and $17.53 \text{ MJ m}^{-2} \text{ day}^{-1}$. The estimates of the asymptotic parameter of the Logistic model, based on Table 3, were $18,420 \text{ kJ m}^{-2}$ for the autumn equinox, $16,010 \text{ kJ m}^{-2}$ for the winter solstice, and $18,790 \text{ kJ m}^{-2}$ for the spring equinox.

In the fit for the autumn equinox, the Logistic model estimated an asymptotic value of $18,240 \text{ kJ m}^{-2}$, an abscissa of the inflection point at 15.43 UTC ($12:26 \text{ BRT}$), and κ of 0.6115 UTC^{-1} ; for the winter solstice, the model estimated an asymptotic value of $16,010 \text{ kJ m}^{-2}$, an abscissa of the inflection point at 15.02 UTC ($12:01 \text{ BRT}$), and κ of 0.7008 UTC^{-1} ; for the spring equinox, the model estimated an asymptotic value of $18,790 \text{ kJ m}^{-2}$, an abscissa

of the inflection point at 14.85 UTC ($11:51 \text{ BRT}$), and κ of 0.6579 UTC^{-1} , as presented in Table 3.

In turn, in the fit for the summer solstice, the Gompertz model estimated an asymptotic value of $23,250 \text{ kJ m}^{-2}$, an abscissa of the inflection point at 13.82 UTC ($10:49 \text{ BRT}$), and κ of 0.3115 UTC^{-1} , as shown in Table 4.

According to a study of data from 2007 to 2021 by Veloso and Souza (2024), the mean global horizontal radiation (Wh m^{-2}) is 220 in March, 175 in June, 240 in September, and 235 in December. According to the work by Ferreira (2006), based on data from February 1926 to October 2006, the maximum mean sunshine duration is 16.5 h in March, 14 h in June, 16 h in September, and 18.5 h in December. The analysis of the data presented by these researchers corroborates the results obtained for the city of Belo Horizonte/MG (Figure 1).

The estimates of the α parameter of the Logistic model fitted to the equinox data are

Table 3. Estimates of the parameters α , β , and κ with the respective lower and upper 95 % confidence limits for the Logistic model fitted to the cumulative global irradiation data throughout the day, for the equinoxes and winter solstice, in the city of Belo Horizonte/MG.

Parameter	Autumn Equinox – March		
	Lower limit	Estimate	Upper limit
α	17,849.8	18,240.0	18,630.2
β	15.29	15.43	15.57
κ	0.5695	0.6115	0.6535
Parameter	Winter Solstice – June		
	Lower limit	Estimate	Upper limit
α	15,759.9	16,010.0	16,260.1
β	14.92	15.02	15.12
κ	0.6592	0.7008	0.7425
Parameter	Spring Equinox – September		
	Lower limit	Estimate	Upper limit
α	18,513.7	18,790.0	19,066.3
β	14.75	14.85	14.95
κ	0.6224	0.6579	0.6934

Source: authors (2023).

Table 4. Estimates of the parameters α , β , κ and with the respective lower and upper limits for the Gompertz model fitted to the cumulative global irradiation data throughout the day, for the summer solstice, in the city of Belo Horizonte/MG, with 95 % confidence intervals.

Parameter	Summer Solstice – December		
	Lower limit	Estimate	Upper limit
α	22,538.40	23,250.00	23,961.60
β	13.6619	13.8200	13.9781
κ	0.2856	0.3115	0.3374

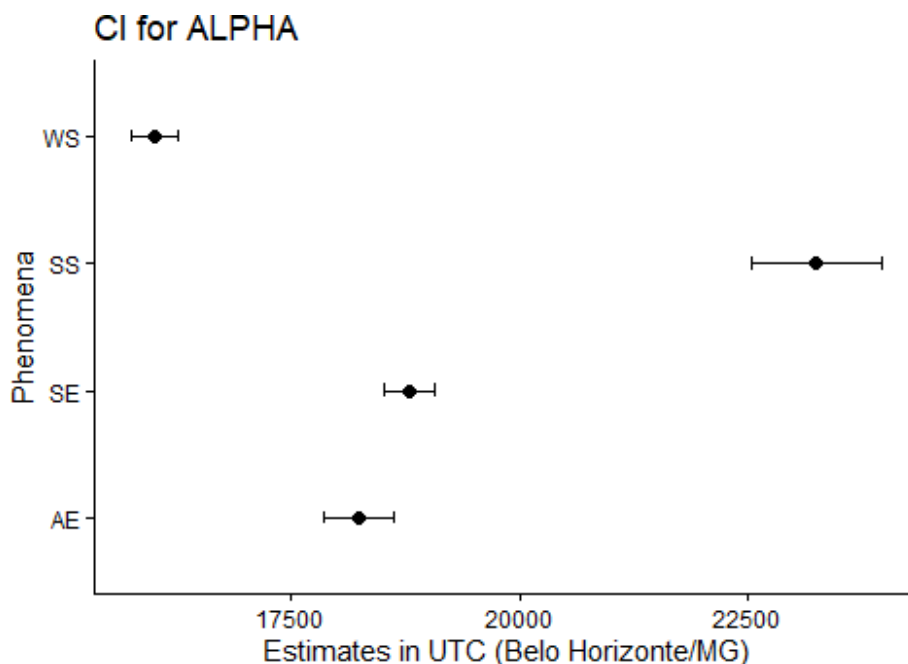
Source: authors (2023).

statistically equivalent, as the confidence intervals for the α parameter overlap, as can be observed in Table 3 and Figure 1 (SILVA *et al.*, 2020a; MIRANDA *et al.*, 2021). However, there is no statistical equivalence for this parameter for the solstices. The observed minimum occurred at the winter solstice and the maximum at the summer solstice (Figure 1). Silva (2008) obtained a similar result for the months of the solstices and equinoxes when analyzing daily exposure to ultraviolet rays: statistically equivalent values in March and September, respectively, 4,324 ($\pm 1,030$) J m⁻² and 4,158 (± 493) J m⁻²;

a minimum in June, 2,509 (± 346) J m⁻²; and a maximum in December, 4,836 ($\pm 1,409$) J m⁻².

In the study by Brandão *et al.* (2016), in the city of Formiga/MG, it was identified that the highest energy intensity occurs in the interval from 12:00 to 13:30. According to Santos and Martins Neto (2016), the maximum availability of solar energy occurs between 11:00 and 14:00 in Belo Horizonte. The models estimate the maxima between 13.82 UTC (10:49 BRT) and 15.43 UTC (12:26 BRT). The estimates of the β parameter for the solstices and the spring

Figure 1. Estimate and 95 % confidence interval for the parameter of the Logistic and Gompertz models fitted to the cumulative daily global irradiation data in the city of Belo Horizonte/MG. SS: Summer Solstice, WS: Winter Solstice, SE: Spring Equinox, AE: Autumn Equinox.



Source: authors (2025).

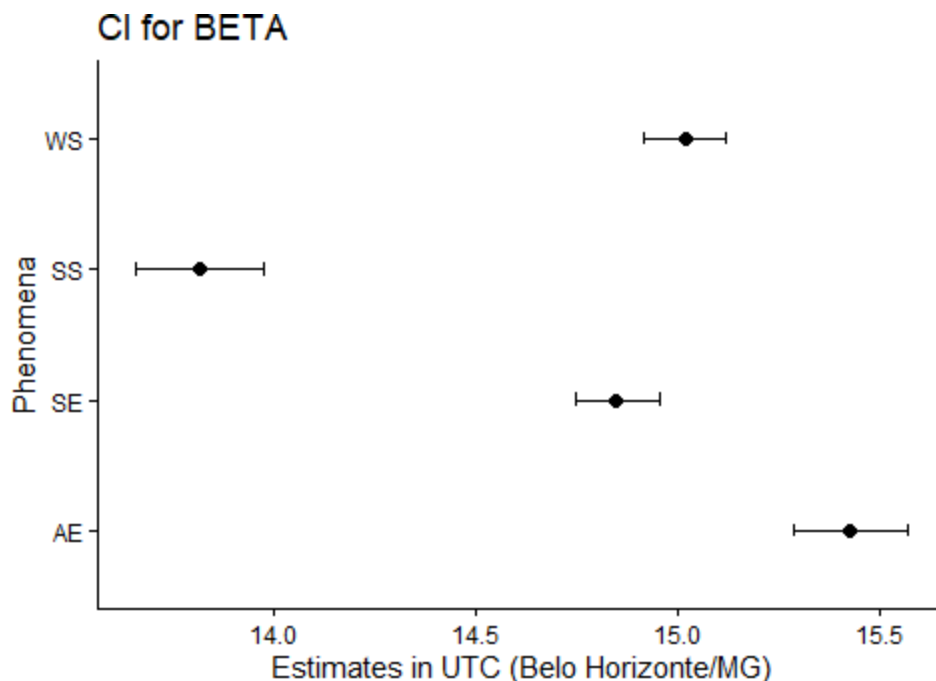
equinox are statistically equivalent. However, the β parameter estimate for the autumn equinox is not equivalent to the others, as shown in Figure 2 and the confidence intervals presented in Tables 3 and 4.

Belo Horizonte has a tropical highland climate, with an average annual temperature of approximately 21 °C and average relative humidity around 72 %. Urban dynamics and topography significantly influence thermal variability and humidification, especially during the pre-dawn and morning hours, which favors fog formation and local saturation (FERREIRA, ASSIS, KATZSCHNER, 2017). According to INMET (2020), relative humidity values in Belo Horizonte tend to be higher in the early morning hours, owing to nocturnal cooling that reduces the air’s capacity to retain water vapor, potentially reaching values close to 100 % on cloudy days or after precipitation events. Precipitation directly affects relative humidity levels, especially in the

early hours of the following day. According to climatological analyses, in Belo Horizonte, the highest average precipitation occurs in summer, during the rainy season—200 mm in December and 250 mm in January (ASSIS, 2012). The distinctive climate characteristics of the summer solstice affect the accumulation of solar irradiation, producing an asymmetry that was captured by the selection of the Gompertz model (Table 2), which accounts for the earlier occurrence of the β parameter (Figure 2 and Table 4).

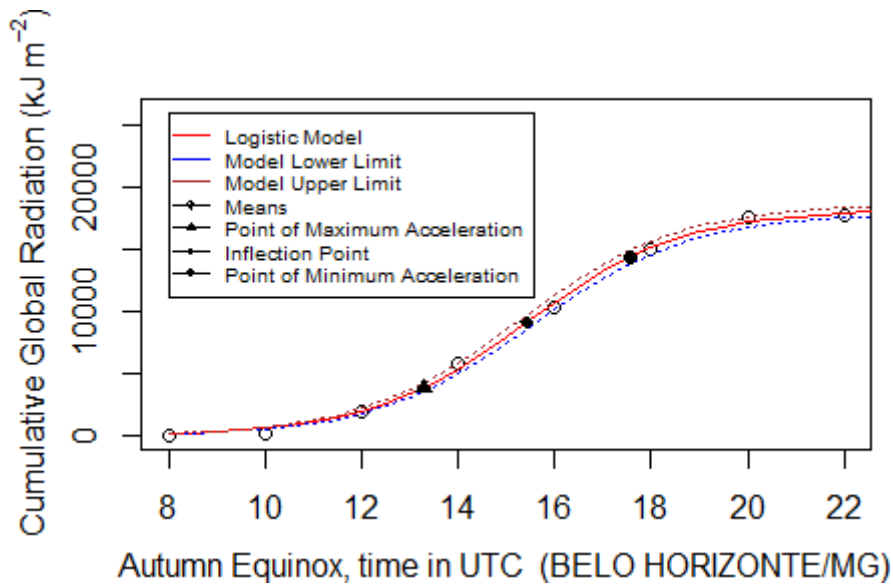
According to the Logistic model fitted to the cumulative means for the autumn equinox, the maximum acceleration occurs at 13:28 UTC with 3,854.57 kJ m⁻² of accumulated energy, and the minimum acceleration occurs at 17:58 UTC with 14,385.43 kJ m⁻² of accumulated energy. Macagnan and Copetti (2007) obtained a mean solar irradiation value of 4,549.4 Wh m⁻² (16,377.84 kJ m⁻²) for March, while the model estimated 18,240 kJ m⁻² (Figure 3).

Figure 2. Estimate and 95 % confidence interval for the β parameter of the Logistic and Gompertz models fitted to the cumulative daily global irradiation data in the city of Belo Horizonte/MG. SS: Summer Solstice, WS: Winter Solstice, SE: Spring Equinox, AE: Autumn Equinox.



Source: authors (2025).

Figure 3. Logistic model fitted with respective 95 % confidence intervals, inflection points, and points of maximum and minimum acceleration of cumulative daily global irradiation, autumn equinox for the period 2018 to 2022, in the city of Belo Horizonte/MG.

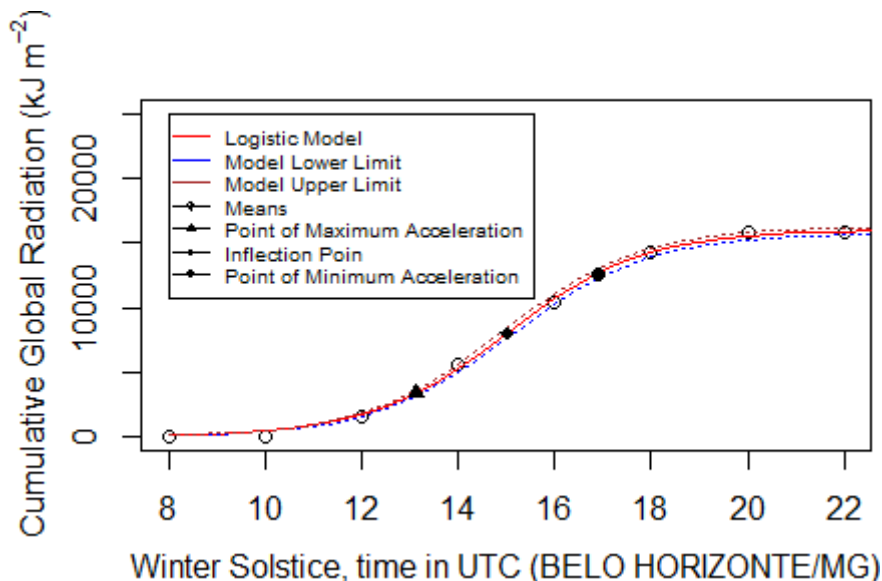


Source: authors (2023).

According to the Logistic model fitted to the cumulative means for the winter solstice, the maximum acceleration occurs at 13:14 UTC with 3,383.31 kJ m⁻² of accumulated energy, and the minimum acceleration occurs at 16:54 UTC

with 12,626.69 kJ m⁻² of accumulated energy. Macagnan and Copetti (2007) obtained a mean solar irradiation value of 3,874.3 Wh m⁻² (13,947.48 kJ m⁻²) for March, while the model estimated 16,010 kJ m⁻² (Figure 4).

Figure 4. Logistic model fitted with respective 95 % confidence intervals, inflection points, and points of maximum and minimum acceleration of cumulative daily global irradiation, winter solstice for the period 2018 to 2022, in the city of Belo Horizonte/MG.



Source: authors (2023).

For the Logistic model fitted to the cumulative means for the spring equinox, the maximum acceleration occurs at 12:51TC with 3,970.79 kJ m⁻² of accumulated energy, and the minimum acceleration occurs at 16:51 UTC with 14,819.21 kJ m⁻² of accumulated energy. Macagnan and Copetti (2007) obtained a mean solar irradiation value of 6,664.8 Wh m⁻² (16,793.28 kJ m⁻²) for March, while the model estimated 18,790 kJ m⁻² (Figure 5).

For the Gompertz model fitted to the cumulative means for the summer solstice, the maximum acceleration occurs at 10:44 UTC with 1,696.06 kJ m⁻² of accumulated energy, and the minimum acceleration occurs at 16:55 UTC with 15,868.47 kJ m⁻² of accumulated energy. Studying the historical series from 1978 to 1988, Macagnan and Copetti (2007) obtained a mean solar irradiation value of 4,335.6 Wh m⁻² (15,608.16 kJ m⁻²) for March, while the model estimated 23,250 kJ m⁻² (Figure 6). The significant difference between these estimates may be associated with differences between the historical series analyzed.

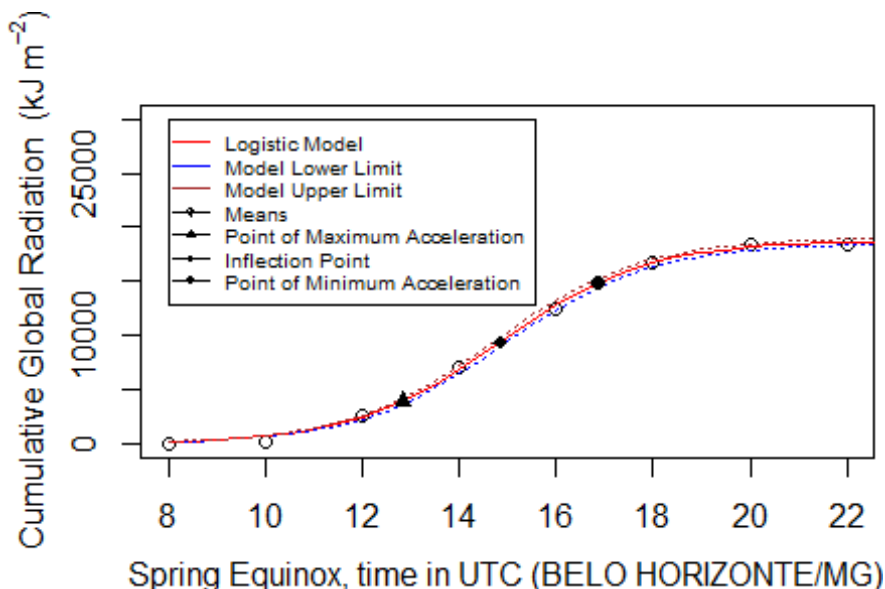
Conclusions

The Logistic model provided the best fit for the equinox and winter solstice data, while the Gompertz model was more adequate for the summer solstice data.

In the fit for the autumn equinox data, the Logistic model estimated an asymptotic value of 18,240 kJ m⁻², with the abscissa of the inflection point at 15:26 UTC (12:26 BRT); for the winter solstice, the model estimated an asymptotic value of 16,010 kJ m⁻², with the abscissa of the inflection point at 15:01 UTC (12:01 BRT); and for the spring equinox, an asymptotic value of 18,790 kJ m⁻², with the abscissa of the inflection point at 14:51 UTC (11:51 BRT). In the fit for the summer solstice data, the Gompertz model estimated an asymptotic value of 23,250 kJ m⁻², with the abscissa of the inflection point at 13:49 UTC (10:49 BRT).

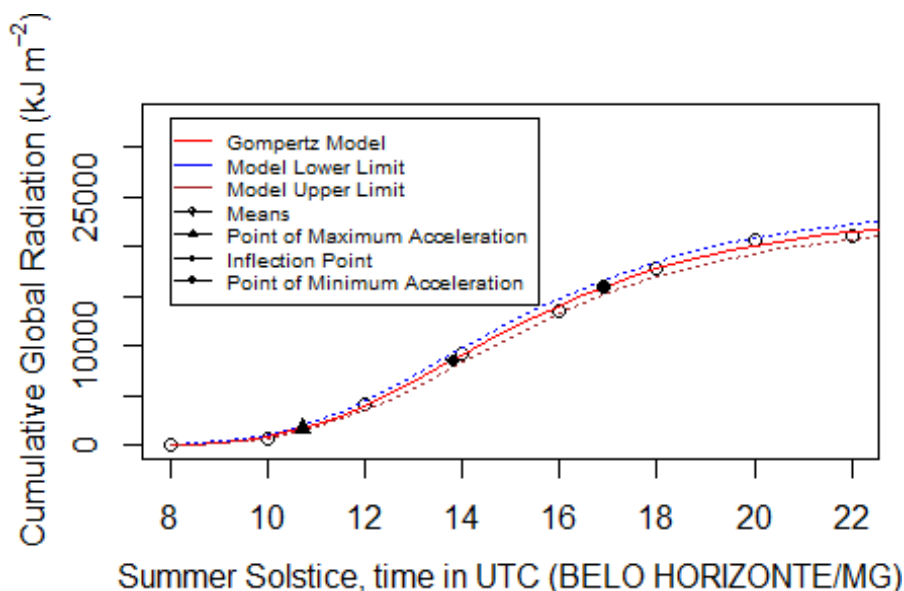
The asymptotic parameter (α) estimates of the Logistic model fitted to the equinox data are statistically equivalent. However, there is no statistical equivalence for this parameter between

Figure 5. Logistic model fitted with respective 95 % confidence intervals, inflection points, and points of maximum and minimum acceleration of cumulative daily global irradiation, spring equinox for the period 2018 to 2022, in the city of Belo Horizonte/MG.



Source: authors (2023).

Figure 6. Gompertz model fitted with respective 95 % confidence intervals, inflection points, and points of maximum and minimum acceleration of cumulative daily global irradiation, summer solstice for the period 2018 to 2022, in the city of Belo Horizonte/MG.



Source: authors (2023).

the solstices. The estimates are consistent with expected values, with the minimum observed at the winter solstice, the maximum at the summer solstice, and values close to the average of the extremes at the equinoxes.

The β parameter (abscissa of the inflection point) estimates for the solstices and the spring equinox are statistically equivalent. However, the estimate for this parameter for the autumn equinox is not equivalent to the others.

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References

ASSIS, W. L. Os climas naturais do município de Belo Horizonte - MG. **Acta Geográfica**, v. 1, p. 115-135, 2012. Disponível em: <http://revista.ufr.br/index.php/actageo/article/view/1098/872>. Acesso em: 14 jun. 2025.

BEDAQUE, P.; BRETONES, P. S. Variação da posição de nascimento do Sol em função da latitude. **Revista Brasileira de Ensino de Física**, v. 38, n. 3, e3307, 2016. DOI: 10.1590/1806-9126. Disponível em: https://www.scielo.br/j/rbef/a/7KxmyjMBkxw5DR_TqMWd9GJR/?format=pdf&lang=pt. Acesso em: 17 jan. 2024.

BRANDÃO, R. R.; PINHEIRO, B. S.; PINTO, P. A.; NOBREGA, R. V. da. Estudo e implementação de sistemas seguidores solar. In: CONGRESSO BRASILEIRO DE ENERGIA SOLAR-CBENS, 2016. **Anais...** São Paulo: Cidade Universitária, 2016. p. 1-8. DOI: 10.59627/cbens.2016.2062. Disponível em: <https://doi.org/10.59627/cbens.2016.2062>. Acessado em: 14 abr. 2025.

Empresa de Pesquisa Energética – EPE. **Balço Energético Nacional 2023**: ano base 2022. Rio de Janeiro: EPE, 2023. Disponível em: <https://www.epe.gov.br/sites-pt/publicacoes-dados-abertos/publicacoes/PublicacoesArquivos/publicacao-748/topico-687/BEN2023.pdf>. Acesso em: 7 fev. 2024.

FERNANDES, T. J.; MUNIZ, J. A.; PEREIRA, A. A.; MUNIZ, F. R.; MUIANGA, C. A. Parameterization effects in nonlinear models to describe growth curves. **Acta Scientiarum. Technology**, v. 37, n. 4, p. 397-402, 2015. Disponível em: <https://repositorio.ufla.br/items/f3611b57-cd55-4689-ac35-4a5058d1853a>. Acesso em: 10 jun. 2025.

FERREIRA, D. G.; ASSIS, E. S. De; KATZSCHNER, L. Construção de um mapa climático analítico para a cidade de Belo Horizonte, Brasil. **URBE. Revista Brasileira de Gestão Urbana**, v. 9, p. 255–270, 2017. Disponível em: <https://doi.org/10.1590/2175-3369.009.SUPL1.A001>. Acesso em: 14 jun. 2025.

FERREIRA, W. P. M. **Radiação solar em Sete Lagoas**. Sete Lagoas: Embrapa Milho e Sorgo, 2006. 21 p. ISSN 1518-4277. Disponível em: <https://www.infoteca.cnptia.embrapa.br/bitstream/doc/490426/1/Doc58.pdf>. Acesso em: 21 mai. 2021.

FRÜHAUF, A. C.; SILVA, É. M. da; GRANATO, D. S.; SILVA, E. M.; MUNIZ, J. A.; FERNANDES, T. J. Descrição do crescimento em altura de clones híbridos de Eucalyptus em região semiárida utilizando modelos não lineares. **Brazilian Journal of Biometrics**, v. 40, p. 138-151, 2022b.

FRÜHAUF, A. C.; SILVA, E. M.; FERNANDES, T. J.; MUNIZ, J. A. Predicting height growth in bean plants using non-linear and polynomial models. **Revista Agrogeoambiental**, v. 13, n. 3, p. 488-497, 2022a. DOI: <https://doi.org/10.18406/2316-1817v13n320211625>.

FRÜHAUF, A. C.; PEREIRA, G. de A.; BARBOSA, A. C. M. C.; FERNANDES, T. J.; MUNIZ, J. A. Nonlinear models in the study of the cedar diametric growth in a seasonally dry tropical forest. **Revista Brasileira de Ciências Agrárias**, v. 15, e8558, 2020.

INMET – Instituto Nacional de Meteorologia. **Climatologia de Belo Horizonte – Normais 1981–2010**. Brasília: 2020. Disponível em: <https://portal.inmet.gov.br/normais>. Acesso em: 14 jun. 2025.

INMET – Instituto Nacional de Meteorologia. **Históricos de dados meteorológicos (2000/2022)**. DF: Brasília, 2023. Disponível em: <https://portal.inmet.gov.br/dadoshistoricos>. Acesso em: 17 jul. 2023.

JANE, S. A.; FERNANDES, F. A.; SILVA, E. M.; MUNIZ, J. A.; FERNANDES, T. J.. Comparison of the polynomial and logistic models on description of pepper growth. **Revista Brasileira de Ciências Agrárias**, v. 14, e7180, 2019.

JANE, S. A.; FERNANDES, F. A.; FERNANDES, T. J.; MUNIZ, J. A. Nonlinear models to describe height and diameter of sugarcane RB92579 variety. **Revista Ciência Agrônômica**, v. 51, e20196660, 2020.

MACAGNAN, M. H.; COPETTI, J. B. Análise estatística de dados de radiação solar no Brasil. In: CONGRESSO BRASILEIRO DE ENERGIA SOLAR - CBENS, 2007. **Anais...** São Paulo, Cidade Universitária, 2007. p. 1-14. DOI: 10.59627/cbens.2007.1324. Disponível em: <https://anaiscbens.emnuvens.com.br/cbens/article/view/1324>. Acesso em: 05 abr. 2025.

MACHADO, L. E. de M. **Modelos não lineares aplicados na descrição do acúmulo da radiação global diária no nível do solo**. 2024. 205 f. Tese (Doutorado em Estatística e Experimentação Agropecuária) – Universidade Federal de Lavras, Lavras, 2024. Disponível em: <http://repositorio.ufla.br/jspui/handle/1/59650>. Acesso em: 09 mai. 2025.

MAZUCHELI, J.; ACHCAR, J.A. Algumas considerações em regressão não linear. **Acta Scientiarum**, v. 24, n. 6, p. 1761-1770, 2002.

Disponível em: <<http://eduem.uem.br/ojs/index.php/ActaSciTechnol/article/view/251>>. Acesso em: 19 out. 2023.

MIRANDA, L. F.; FRÜHAUF, A. C.; LIMA, K. P.; SILVA, E. M.; MUNIZ, J. A. Nonlinear models to describe the growth of *Jatropha* plant (*Jatropha curcas* L.). **Revista Ciência Agronômica**, v. 52, e20207602, 2021.

NEGRÃO, O. B. M. Movimentos da terra e clima. **Ciências em Foco**, v. 1, n. 1, p. 1-3, 2008. Disponível em: <https://econtents.bc.unicamp.br/inpec/index.php/cef/article/view/9164>. Acesso em: 21 jun. 2021.

OLIVEIRA, W. S. J. de; SILVA, C. A.; MUNIZ, J. A.; SAVIAN, T. V. Mineralização de nitrogênio em latossolos adubados com resíduos orgânicos. **Revista Brasileira de Ciência do Solo**, v. 37, p. 715-725, 2013.

PAIVA, E. C. **Energia Solar, no topo da atmosfera, ao nível do solo e simulações**. **Revista Científica Multidisciplinar Núcleo do Conhecimento**, Ano 3, Ed. 12, v. 7, p. 51-77, 2018. ISSN: 2448-0959. Disponível em: <https://www.nucleodoconhecimento.com.br/engenharia-agricola/no-topo-da-atmosfera>. Acesso em: 07 jan. 2024.

PAULA, G. S.; SILVA, E. M.; FRÜHAUF, A. C.; SILVA, E. M.; MUNIZ, J. A.; FERNANDES, T. J. Modeling the amount of mineralized carbon from swine manure and wheat straw. **Revista Agrogeoambiental**, v. 12, n. 3, p. 60-77, 2020. DOI: <http://dx.doi.org/10.18406/2316-1817v12n320201466>.

PAULA, G. S.; SILVA, E. M.; FURTADO, T. D. R.; FRÜHAUF, A. C.; MUNIZ, J. A. Comparison of nonlinear models for the description of carbon mineralization in soils treated with pig slurry. **Revista Agrogeoambiental**, v. 11, n. 4, p. 82-95, 2019. DOI: <http://dx.doi.org/10.18406/2316-1817v11n420191412>.

PRADO, T. K. L. do; SAVIAN, T. V.; FERNANDES, T. J.; MUNIZ, J. A. Study on the growth curve of the internal cavity of Dwarf green coconut fruits. **Agronomic Science Journal**, v. 51, e20154591, 2020.

R CORE TEAM. **R: A language and environment for statistical computing**. Viena: R Foundation for Statistical Computing, 2025. Programa de computador. Disponível em: <https://www.r-project.org/>. Acesso em: 8 fev. 2025.

RAMOS; J. P. A.; VIANNA, M. S.; MARIN, F. R. Estimativa da radiação solar global baseada na amplitude térmica para o Brasil. **Agrometeoros**, v. 26, n. 1, p. 37-51, 2018. Disponível em: <https://repositorio.usp.br/directbitstream/e40185d4-355f-418c-b041-c0089b5da9fb/3027481>. Acesso em: 05 abr. 2025.

RIBEIRO, T. D.; MATTOS, R. W. P.; MORAIS, A. R. de; MUNIZ, J. A. Description of the growth of pequi fruits by nonlinear models. **Revista Brasileira de Fruticultura**, v. 40, e-949, 2018.

SANTOS, P. R. S.; MARTINS NETO, J. H. Investigação das curvas de demanda elétrica em setor comercial e residencial devido à implementação de sistemas fotovoltaicos conectados à rede. In.: CONGRESSO BRASILEIRO DE ENERGIA SOLAR, 2016. **Anais...** São Paulo: Cidade Universitária, 2016. p. 1–8. DOI: 10.59627/cbens.2016.2057. Disponível em: <https://anaiscbens.emnuvens.com.br/cbens/article/view/2057>. Acesso em: 14 abr. 2025.

SILVA, A. A. Medidas de radiação solar ultravioleta em Belo Horizonte e saúde pública. **Revista Brasileira de Geofísica**, v. 26, n. 4, p. 417–425, 2008. Disponível em: <https://doi.org/10.1590/S0102-261X2008000400003>. Acesso em: 17 mai. 2025.

SILVA, E. M. da; TADEU, M. H.; SILVA, V. F. da; PIO, R.; FERNANDES, T. J.; MUNIZ, J.

A. Description of blackberry fruit growth by nonlinear regression models. **Revista Brasileira de Fruticultura**, v. 42, n. 2, e-177, 2020a. DOI: <https://doi.org/10.1590/0100-29452020177>

SILVA, E. M.; FRÜHAUF, A. C.; JANE, S. A.; SILVA, É, M da; MUNIZ, J. A.; FERNANDES, T. J. Comparison of nonlinear models in the description of carbon mineralization in litter soil. **Revista Agrogeoambiental**, v. 12, n. 4, p. 58-69, 2020b. DOI: <http://dx.doi.org/10.18406/2316-1817v12n420201484>.

SILVA, E. M.; FURTADO, T. D. R.; FERNANDES, J. G. F.; MUNIZ, J. A. Description of the carbon mineralization of swine manure and oat straw in the soil through nonlinear models. **Revista Agrogeoambiental**, v. 11, n. 2, p. 71-86, 2019a. DOI: <http://dx.doi.org/10.18406/2316-1817v11n220191299>.

SILVA, E. M.; JANE, S. A.; FERNANDES, F. A.; SILVA, É, M.; MUNIZ, J. A.; FERNANDES, T. J. Stanford & Smith nonlinear model in the description of CO₂ evolved from soil treated with swine manure: maximum entropy prior. **Acta Scientiarum. Technology**, v. 45, n. 1, p. 1-11, 2022. DOI: <https://doi.org/10.4025/actascitechnol.v45i1.56360>.

SILVA, E. M.; MUNIZ, J. A.; FERNANDES, T. J. Gompertz model describing CO₂ evolved from legumes in the soil: bayesian approach with maximum entropy prior. **Acta Scientiarum. Technology**, v. 47, n. 1, p. 1-10, 2025. DOI: <https://doi.org/10.4025/actascitechnol.v47i1.70163>.

SILVA, E. M.; SILVEIRA, S. C.; FURTADO, T. D. R.; MUNIZ, J. A. Fitting of decomposition of sewage sludge and oat straw by nonlinear models. **Revista Agrogeoambiental**, v. 11, n. 2, p. 18-28, 2019b. DOI: <http://dx.doi.org/10.18406/2316-1817v11n220191287>.

SILVA, W da S. e; FERNANDES, F. A.; MUNIZ, F. R. ; MUNIZ, J. A.; FERNANDES, T. J.. *Eucalyptus grandis* x *Eucalyptus urophylla* growth curve in different site classifications, considering residual autocorrelation. **Revista Brasileira de Biometria**, v. 39, p. 122-138, 2021.

SILVEIRA, S. C.; MUNIZ, J. A.; SOUSA, F.A.; CAMPOS, A. T. Modelos não lineares ajustados à produção acumulada de biogás provenientes de camas sobrepostas de suínos. **Revista Agrogeoambiental**, v. 10, p. 91-103, 2018.

SOUZA, I. F.; KUNZLE NETO, J. E.; MUNIZ, J. A.; GUIMARÃES, R. M.; SAVIAN, T. V.; MUNIZ, F. R. Fitting nonlinear autoregressive models to describe coffee seed germination. **Ciência Rural**, v. 44, p. 2016-2021, 2014.

VELOSO, A. C. de O.; SOUZA, R. V. G. de. Mudanças climáticas e consumo de energia de edifícios de escritório com ventilação híbrida em Belo Horizonte. In: ENCONTRO NACIONAL DE TECNOLOGIA DO AMBIENTE CONSTRUÍDO, 20., 2024. **Anais...** Porto Alegre: ANTAC, 2024. p. 1–10. DOI: 10.46421/entac.v20i1.5875. Disponível em: <https://eventos.antac.org.br/index.php/entac/article/view/5875>. Acesso em: 10 mai. 2025.

VILELA, M. S. L.; SILVA, E. M.; FRÜHAUF, A. C.; SILVA, É. M.; MUNIZ, J. A.; FERNANDES, T. J. Comparison of nonlinear models for the description of carbon mineralization in degraded pasture soil and in soils with plant cover. **Revista Agrogeoambiental**, v. 14, n. 1, p. 1-12, 2022. DOI: <https://doi.org/10.18406/2316-1817v14n120221680>.